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PATH DECOMPOSITION OF RESTRICTED SUPER LINE GRAPH OF PATH GRAPH

Dr. Latha Devi Puli
Assistant Professor, Department of Mathematics, Government First grade College, Yelahanka,,
Bangalore- 560064 Karnataka, drlathadevip@gmail.com

Kousalya
Assistant Professor, Department of Mathematics, Government First grade College,
Malleshwaram, 18th cross, Bangalore- 560012 Karnataka
kousalyaharish2015@gmail.com

Abstract:
A decomposition of a graph G is a collection ψ of graphs $H_1, H_2, ...H_r$ of G such that every edge of G belongs to exactly one of H_i. If each H_i is a path, then ψ is called path decomposition of G. In this paper we discussed path decomposition of restricted super line graph of index 2 of G when G is isomorphic to Path graph

Key words: Path decomposition, restricted super line graph.

AMS Subject Classification: 05C70

1.Introduction:

The fundamental concept of path decomposition in graphs as introduced by Harary [7] continues to be of interest to researchers due to its wide range of applications in real life. The study on decomposition in paths helps us to understand, analyse and design networks effectively. Research in this area helps us analyse problems in transportation, distribution, designing, communication, team formation and event management. Extensive research has been dedicated to the study of various types of decompositions and related parameters in [1, 2, 3, 4, 6] in context of paths, cycles and common vertices between the paths.

Graph decomposition problems rank among the most prominent areas of research in graph theory and combinatorics and further it has numerous applications in various fields such as networking, block designs, and bioinformatics. A path decomposition of a graph G is a partition of edges into subgraphs H_i each of which is a path or a union of paths (linear forests). Various types of decompositions and corresponding parameters have been studied by several authors by imposing different conditions on H_i. Some of such decompositions are path decomposition, cyclic decomposition, acyclic decomposition etc.

Let G = (V, E) be a simple graph without loops or multiple edges. A path is a walk where $v_i \neq v_j, i \neq j$. In other words, a path is a walk that visits each vertex at most once. A decomposition of a graph G is a collection of edge-disjoint subgraphs $G_1, G_2, \ldots G_n$ of G such that every edge of G belongs to exactly one $G_i, 1 \leq i \leq m$. $E(G) = E(G_1) \cup E(G_2) \ldots E(G_m)$. If every graph G_i is a path then the decomposition is called a path decomposition. All graphs considered in this paper are simple graphs. Restricted Super line graph of index r of a graph G, denoted by $RL_r(G)$, is introduced by Manjula and Sooryanarayana [8].It is a modification of the concept of the super line graph L(G) introduced by Bagga [5]. The vertices of $RL_r(G)$ are the r-element subsets of E(G) and two vertices S and T are adjacent if there exists exactly one pair of edges, one from each of the sets S and T, which are adjacent in G.

We need a few observations to obtain the result. First consider an $n \times m$ array $R_{n,m}$ of points where a point in i^th^ row and j^th^ column is identified with the edge (x_i, y_j) of a graph G on which the vertex sets $\{x_1, x_2, \ldots x_n, y_1, y_2, \ldots y_m\}$ are defined. Any path on the points $R_{n,m}$ with properties (i) travels only along rows .(ii) uses at most two points from any row or column and (iii) whose end points does not lie in the same row or column defines a unique path in G. If a path with (i),(ii)&(iii) in $R_{n,m}$ uses N points then the corresponding path in G uses exactly N-1 edges and has no repeated vertices.

Now the problem of decomposing $RL_2(G)$ into paths $P_i, i \leq 2n-10$ reduces to covering of $RL_2(G)$ with paths using different points and each satisfying conditions (i),(ii)&(iii).

2.Main results

Lemma 2.1: $\psi(RL_2(P_n) = 2P_{10} \cup 3P_9 \cup 3P_7 \cup 3P_6 \cup 2P_5 \cup 3P_4 \cup 5P_3 \cup 12P_2 \;\; for\; n = 8$

Proof: Let $e_i \in E(P_8), 1 \leq i < 8$ Then the path decomposition of $RL_2(P_8)$ is given as

$\psi(RL_2(P_8) = e_1e_3, e_1e_4, e_1e_2, e_2e_4 --P_4;\; e_2e_4, e_1e_4, e_3e_4, e_1e_3, e_4e_5, e_2e_3, e_3e_5, e_2e_5, e_1e_5 --P_9$
$e_3e_4, e_1e_2, e_1e_5, e_1e_4, e_4e_5, e_2e_4, e_2e_5 --P_7; e_3e_5, e_1e_2, e_2e_5, e_2e_3, e_1e_5, e_3e_4 --P_6; e_1e_5, e_4e_5, e_2e_5 --P_3;$
$e_1e_6, e_1e_2, e_2e_6, e_1e_4, e_3e_6, e_2e_3, e_4e_6, e_3e_4, e_5e_6, e_2e_4 --P_{10};, e_1e_2, e_3e_6, e_3e_4, e_2e_6, e_2e_3, e_1e_6, e_2e_4 --P_7;$
$e_4e_6, e_1e_3 --P_2;\; e_5e_6, e_1e_4 --P_2;\; e_1e_5, e_1e_6, e_3e_5, e_3e_6, e_1e_5, e_5e_6, e_2e_5, e_2e_6, e_4e_5 --P_9;$
$e_5e_6, e_3e_5 --P_2; e_5e_6, e_1e_6, e_2e_6, e_3e_6, e_4e_6 --P_5;\; e_2e_6, e_5e_6 --P_2; e_3e_6, e_5e_6 --P_2; e_5e_7, e_1e_4 --P_2;$
$e_1e_7, e_1e_2, e_2e_7, e_1e_4, e_3e_7, e_2e_3, e_4e_7, e_3e_4, e_5e_7, e_2e_4 --P_{10}; e_5e_7, e_2e_4 --P_2; e_4e_7, e_6e_7 --P_2; e_5e_6, e_2e_4 --P_2;$
$e_3e_7, e_1e_2, e_3e_4, e_2e_7 --P_4; e_2e_3, e_1e_7, e_3e_4 --P_3; e_4e_7, e_1e_3 --P_2;\; e_1e_7, e_2e_5, e_3e_7, e_4e_5, e_6e_7, e_3e_5,$
$e_2e_7, e_1e_5, e_4e_7 --P_9;\; e_1e_5, e_6e_7, e_2e_5, e_4e_7, e_4e_5, e_5e_7 --P_6;\; e_1e_6, e_1e_7, e_3e_6, e_3e_7, e_5e_6, e_2e_7, e_2e_6 --P_7;$
$e_2e_7, e_4e_{6,}\; e_4e_7 --P_3; e_4e_6, e_1e_{7,}\; e_5e_6 --P_3; e_3e_7, e_1e_6, e_2e_6, e_6e_7, e_3e_6 --P_5;$
$e_1e_6, e_6e_{7,}\; e_4e_6 --P_3; e_6e_7, e_1e_7, e_2e_7, e_3e_7, e_4e_7, e_5e_7 --P_6; e_2e_7, e_6e_7 --P_2; e_3e_7, e_6e_7 --P_2$

Thus the edges of $RL_2(P_8)$ can be decomposed into

$\psi(RL_2(P_n)) = 2P_{10} \cup 3P_9 \cup 3P_7 \cup 3P_6 \cup 2P_5 \cup 3P_4 \cup 5P_3 \cup 12P_2$

Theorem 2.2:

$$\psi(RL_2(P_n)) = \psi(RL_2(P_8)) \cup (n-8)(P_{10} \cup P_7 \cup 2P_2) \bigcup_{i=0}^{n-9} (n-8-i)P_{11+2i} \cup \frac{(n-8)(n-9)}{2}(P_6 \cup 2P_3)$$

$$\bigcup_{i=9}^{n} \left(P_{2i-9} \cup P_{2i-10} \cup P_{2i-13} \cup P_{2i-14}\right) \bigcup_{i=10}^{n} P_{2n-16} \bigcup_{i=11}^{n} P_{2n-18} \bigcup_{i=112}^{n} P_{2n-20} \dots\dots\dots \cup (P_6 \cup P_4) \cup X$$

where $X = \begin{cases} P_3 \cup 2P_2 \text{ for even } n \\ 2P_3 \text{ forodd } n \end{cases} \text{for } n \geq 9$

Proof : Let $e_i \in E(P_n), 1 \leq i \leq n-1$. Then $\left|V((RL_2(P_n))\right| = \frac{(n-1)(n-2)}{2}$.

The edges between these vertices can be decomposed into paths to get the path covering as follows.

$$\psi(RL_2(P_n)) = \psi(RL_2(P_8)) \cup \left\{ \begin{array}{l} \psi\left(E\begin{pmatrix} e_i, e_j \,\&\, e_k, e \\ 1 \leq i < j \\ 8 \leq j < n-1 \\ 1 \leq k < l \leq 4 \end{pmatrix}\right) \cup \psi\left(E\begin{pmatrix} e_i, e_j \,\&\, e_k, e_5 \\ 1 \leq i < j \,\&\, k < 5 \\ 8 \leq j < n-1 \end{pmatrix}\right) \cup \psi\left(E\begin{pmatrix} e_i, e_j \,\&\, e_k, e_6 \\ 1 \leq i < j \,\&\, k < 6 \\ 8 \leq j < n-1 \end{pmatrix}\right) \cup \\ \psi\left(E\begin{pmatrix} e_i, e \,\&\, e_k, e_7 \\ 1 \leq i < j \,\&\, k < 7 \\ 9 \leq j < n-1 \end{pmatrix}\right) \cup \psi\left(E\begin{pmatrix} e_i, e_j \,\&\, e_k, e_8 \\ 1 \leq i < j \,\&\, k < 8 \\ 10 \leq j < n-1 \end{pmatrix}\right) \dots\dots \cup \psi\left(E\begin{pmatrix} e_i, e_{n-1} \,\&\, e_j, e_{n-2} \\ 1 \leq i < n-1 \\ 1 \leq j < n-2 \end{pmatrix}\right) \\ \cup \psi\left(E\begin{pmatrix} e_i, e_{n-1} \,\&\, e_i, e_{n-1} \\ 1 \leq i, j < n-1 \end{pmatrix}\right) ------(1) \end{array} \right\}$$

The edge decomposition between the vertices $e_i, e_j \,\&\, e_k, e_l, 1 \leq i < j, 8 \leq j < n-1, 1 \leq k < l \leq 4$ is given as shown below.

Table 2.1	$e_{1,2}$	$e_{1,3}$	$e_{1,4}$	$e_{2,3}$	$e_{2,4}$	$e_{3,4}$
$e_{1,j}$	1			1	1	
$e_{2,j}$	1		1	1		1
$e_{3,j}$	1		1	1		1
$e_{4,j}$		1		1		1
$e_{5,j}$			1		1	1
$e_{6,j}$						
$e_{7,j}$ . . .						
$e_{j-1,j}$						

Table 2.2	$e_{1,5}$	$e_{2,5}$	$e_{3,5}$	$e_{4,5}$
$e_{1,j}$		1		
$e_{2,j}$	1		1	
$e_{3,j}$	1	1		1
$e_{4,j}$		1		1
$e_{5,j}$				1
$e_{6,j}$	1	1	1	1
$e_{7,j}$ . . .				
$e_{j-1,j}$				

For convenience sake while constructing the table the vertex $e_{i,}e_j$ is represented by $e_{i,j}$.

Here the paths are

$e_{1,j}, e_{1,2}, e_{2,j}, e_{1,4}, e_{3,j}, e_{2,3}, e_{4,j}, e_{3,4}, e_{5,j}, e_{2,4} --- P_{10}$

$e_{2,4}, e_{1,j}, e_{2,3}, e_{2,j}, e_{2,3}, e_{3,4}, e_{3,4}, e_{3,j}, e_{1,2} --- P_9$ $e_{4,j}, e_{1,3} --- P_2$ $, e_{5,j}, e_{1,4} --- P_2$ $, 2 \le j \le n$

Further for $i \ge 6$,none of the vertex $e_{i,j}$ is adjacent to $e_{k,l}, 1 \le k, l < 4$.So $\forall j \ge 7$ and the edges between these vertices can be decomposed into $P_{10} \cup P_9 \cup 2P_2$ and for a path P_n there exists $(n-8)$ such path decompositions. Thus the edges between these vertices can be decomposed into $(n-8)(P_{10} \cup P_9 \cup 2P_2)$ paths .

The edge decomposition between the vertices $e_i e_j$ & $e_k e_l, 1 \le i < j, 8 \le j < n-1$ & $k < 5$ is as shown in table 2.2.

For $i \ge 6$,none of the vertex $e_{i,j}$ is adjacent to $e_{k,l}, 1 \le k, < 5$.So $\forall j \ge 7$,the the edges between these vertices can be decomposed into $P_9 \cup P_6$ and for a path P_n there exists $(n-8)$ such path decompositions. Thus the edges between these vertices can be decomposed into $(n-8)(P_9 \cup P_6)$ paths.

Table 2.3	$e_{1,7}$	$e_{2,7}$	$e_{3,7}$	$e_{4,7}$	$e_{5,7}$	$e_{6,7}$
$e_{1,j}$		1				
$e_{2,j}$	1		1			
$e_{3,j}$		1		1		
$e_{4,j}$			1		1	
$e_{5,j}$				1		1
$e_{6,j}$	1	1	1	1		1
$e_{7,7}$						1
$e_{8,j}$	1	1	1	1	1	1
$e_{9,j}$ . . .						
$e_{j-1,j}$						

Further the edge decompositions into paths between the vertices $e_ie_j \,\&\, e_ke_l$, $1 \le i < j, 8 \le j < n-1 \,\&\, k < 6$ are given as follows.

$e_{5,j}, e_{1,6}, e_{2,j}, e_{3,6}, e_{4,j}, e_{5,6}, e_{7,j}, e_{4,6}, e_{3,j}, e_{2,6}, e_{1,j} --- P_{11}$

$e_{1,6}, e_{7,j}, e_{2,6}, e_{5,j}, e_{5,6}, e_{6,j} --- P_6; e_{5,j}, e_{3,6}, e_{7,j} --- P_3$

Further for $i \ge 8$,none of the vertex $e_{i,j}$ is adjacent to $e_{k,6}, 1 \le k, l < 6$ .and there exists (n-8) such path decompositions. Thus there are $(n-8)(P_{11} \cup P_6 \cup P_3)$ path decompositions between these vertices.

The edge decomposition between the vertices $e_ie_j \,\&\, e_ke_l, 1 \le i < j, 9 \le j < n-1 \,\&\, k < 7$ is given as shown below and for $i \ge 9$, none of the vertex $e_{i,7}$ is adjacent to $e_{i,j}, 1 \le i < j$ .and there exists (n-9) such path decompositions. Thus there are $(n-9)(P_{13} \cup P_6 \cup 2P_3)$ path decompositions between these vertices. By proceeding in the same way path decompositions between the vertices $e_ie_{n-1} \,\&\, e_ke_{n-3}, 1 \le i < n-1, 1 \le k < n-3$ is given by $P_{2n-7} \cup P_6 \cup (n-8)P_3$

Further edge decomposition between the vertices $e_ie_j \,\&\, e_ke_{n-2}, 1 \le i < j \le n-1 \,\&\, 1 \le k < n-2$ into paths for even n are given as given in tables. Similarly we can decompose into paths for odd n .

Path decomposition between the vertices $e_i e_{n-1}$ & $e_j e_{n-2}, 1 \le i < n-1, 1 \le j < n-2$ is given as in table 2.4 are

$$\left(P_{2n-9} \cup P_{2n-11} \cup P_{2n-13} \cup .....P_{11} \cup P_9\right) \cup \left(P_{2n-10} \cup P_{2n-12} \cup P_{2n-14} \cup .....P_{10} \cup P_8\right) \cup$$
$$\left(P_{2n-13} \cup P_{2n-15} \cup P_{2n-17} \cup .....P_7 \cup P_5\right) \cup \left(P_{2n-14} \cup P_{2n-16} \cup P_{2n-18} \cup .....P_6 \cup P_4\right) \cup ....$$
$$\cup \left(P_8 \cup P_6 \cup P_4\right) \cup \left(P_6 \cup P_4\right) \cup P_4 \cup X \quad where \quad X = \frac{3n-25}{2} P_3 \cup (n-7)P_2 \; for \; odd \; n$$
$$= \frac{3n-25}{2} P_3 \cup (n-7)P_2 \; for \; even \; n$$

Thus there are

$$\bigcup_{i=9}^{n} P_{2i-9} \bigcup_{i=9}^{n} P_{2i-10} \bigcup_{i=9}^{n} P_{2i-13} \bigcup_{i=9}^{n} P_{2i-14} \bigcup_{i=10}^{n} P_{2i-16} \bigcup_{i=11}^{n} P_{2i-18} \bigcup_{i=12}^{n} P_{2n-20} .....(P_6 \cup P_4) \cup P_4 \cup X$$ paths between the above mentioned vertices.

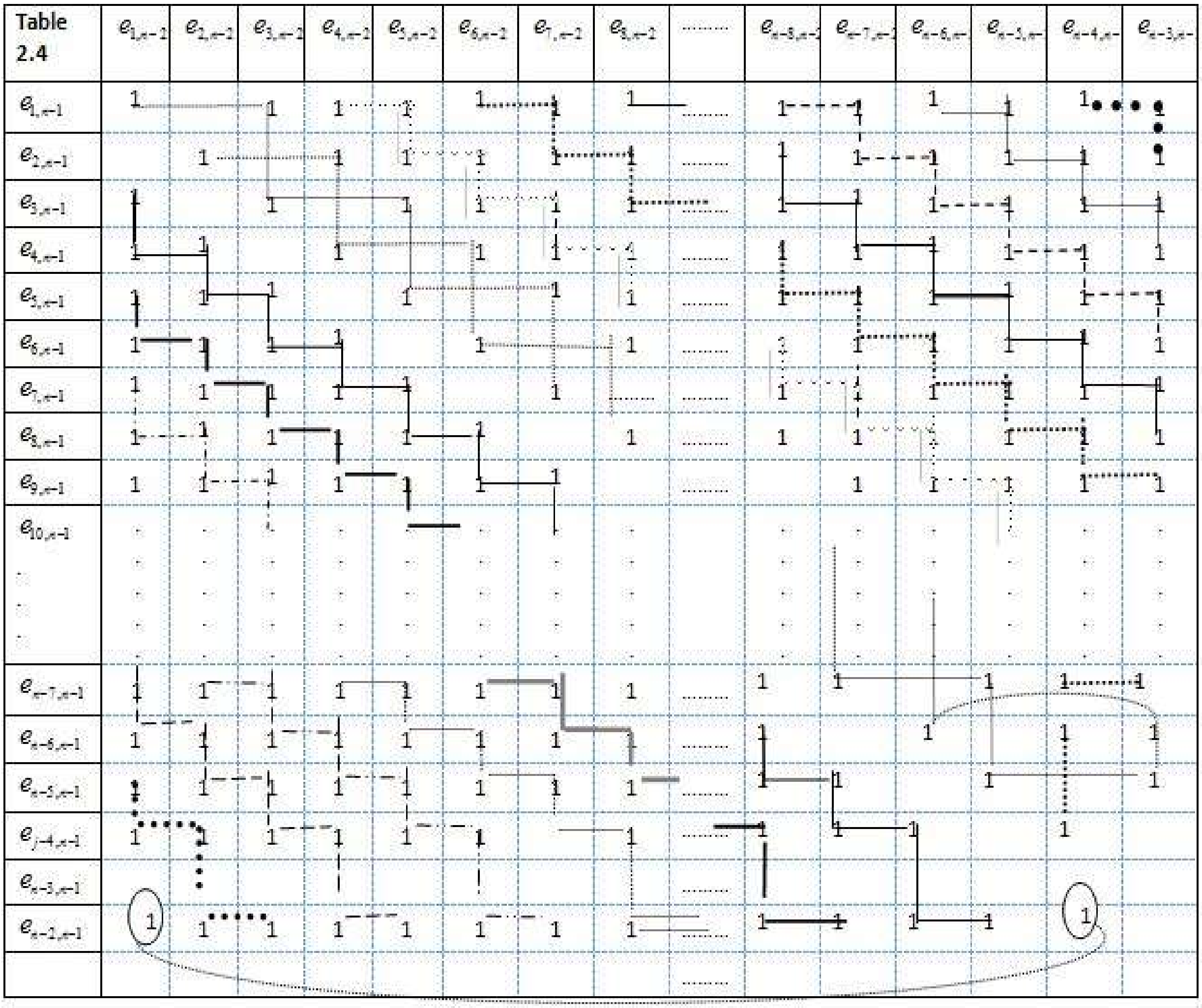
Path decomposition between the vertices $e_i e_{n-1}$ & $e_i e_{n-1}, 1 \le i < n-1$ is given as

$$e_{n-2}e_{n-1}; e_1 e_j; e_1 e_j; e_1 e_j; e_1 e_j; .....e_{n-3}e_{n-1} ----P_{n-2}; \quad e_k e_j, e_{j-1}e_j ----P_2 \; for \; 2 \le k \le j-3$$

Thus there are $P_{n-2} \cup (n-5)P_2$ between the above mentioned vertices.

So by using (1) path decomposition of $RL_2(P_n) \; for \; n \ge 9$ is given as

$$\psi(RL_2(P_n) = \psi(RL_2(P_8) \cup (n-8)\left(P_{10} \cup P_7 \cup 2P_2\right) \bigcup_{i=0}^{n-9} (n-8-i)P_{2i+11} \cup \frac{(n-8)(n-9)}{2}(P_6 \cup 2P_3)$$
$$\bigcup_{i=9}^{n}\left(P_{2i-9} \cup P_{2i-10} \cup P_{2i-13} \cup P_{2i-14} \cup P_{2i-10} \cup P_{2i-10}\right) \bigcup_{i=10}^{n} P_{2i-16} \bigcup_{i=11}^{n} P_{2i-18} \bigcup_{i=12}^{n} (P_{2i-20}) ....$$
$$\cup (P_6 \cup P_4) \cup P_4 \cup X$$



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